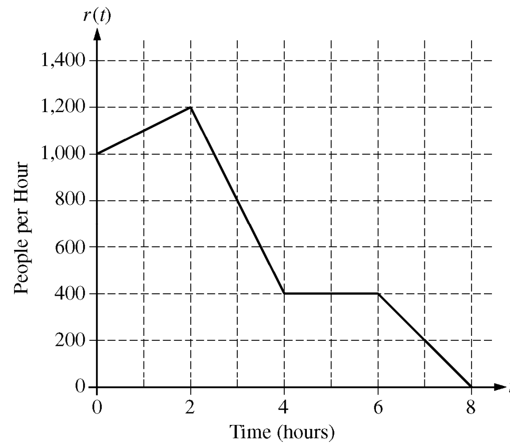
AP® CALCULUS AB
2010 SCORING GUIDELINES

Question 3



There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.

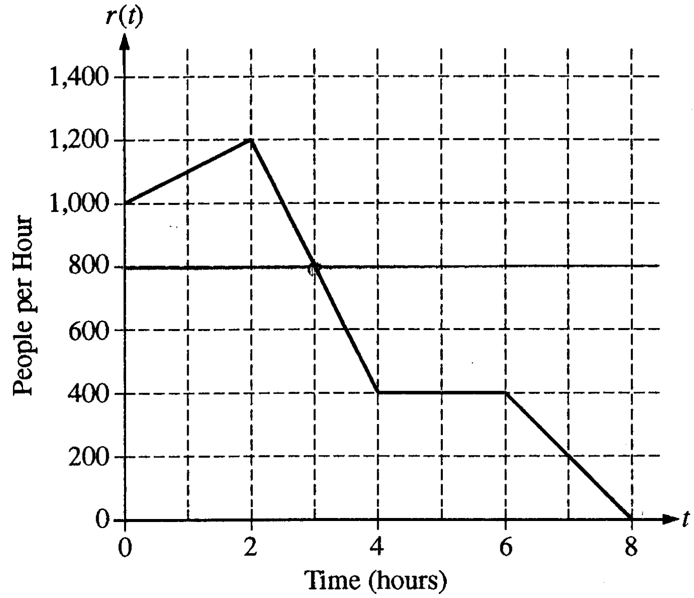
(a) How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

(b) Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.

(c) At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.

(d) Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.

(a) $\int_0^3 r(t)\,dt = 2\cdot\frac{1000+1200}{2} + \frac{1200+800}{2} = 3200$ people	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(b) The number of people waiting in line is increasing because people move onto the ride at a rate of 800 people per hour and for $2 < t < 3$, $r(t) > 800$.	1 : answer with reason
(c) $r(t) = 800$ only at $t = 3$ For $0 \le t < 3$, $r(t) > 800$. For $3 < t \le 8$, $r(t) < 800$. Therefore, the line is longest at time $t = 3$. There are $700 + 3200 - 800\cdot 3 = 1500$ people waiting in line at time $t = 3$.	$3 : \begin{cases} 1 : \text{identifies } t = 3 \\ 1 : \text{number of people in line} \\ 1 : \text{justification} \end{cases}$
(d) $0 = 700 + \int_0^t r(s)\,ds - 800t$	$3 : \begin{cases} 1 : 800t \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$



Work for problem 3(a)

Initially 700 ppl present

$\int_0^3 r(t)\,dt$ ← amount arrived from t=0 to t=3

$= \frac{(1000+1200)\times 2}{2} + \frac{(800+1200)\times 1}{2}$

= 3200 ppl

Work for problem 3(b)

Between t=2 and t=3

The rate of ppl arriving is greater than the rate in which ppl move onto the ride

Therefore the number of ppl waiting in line is increasing between t=2 and t=3.

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Continue problem 3 on page 9.

Work for problem 3(c)

The line is the longest at t=3 since from t=0 to t=3 r(t) > the rate ppl move onto the ride. From t=3 to t=8, r(t) < the rate ppl move onto the ride so the lineup will be shorter

Amount of ppl in line

$700 + \int_0^3 r(t)\,dt - (800 \times 3)$

$= 3900 - 2400$

$= 1500$ ppl in line at $t = 3$

Work for problem 3(d)

$700 + \int_0^t r(x)\,dx - \int_0^t 800\,dx = 0$

or

$700 + \int_0^t (r(x) - 800)\,dx = 0$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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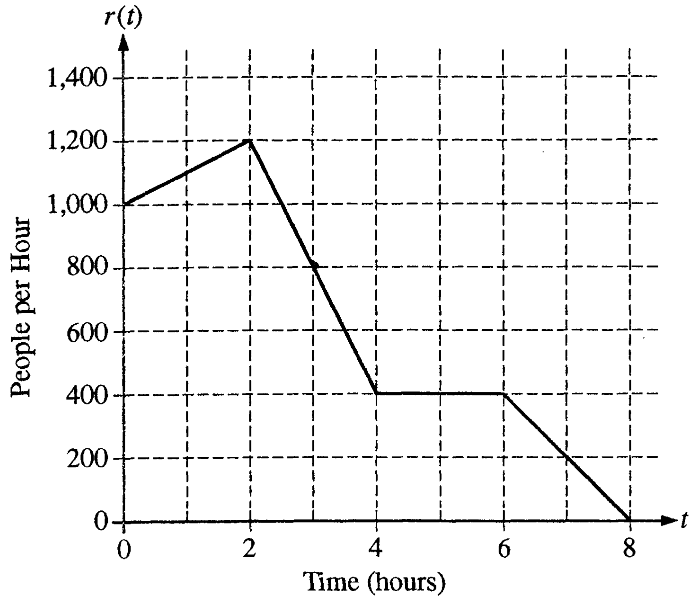

Work for problem 3(a)

$\int_0^3 r(t)\,dt = 3{,}200$

Work for problem 3(b)

increasing because the people boarding the ride remains constant while the people arriving at the line is still greater than the 800 people per hour boarding

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Continue problem 3 on page 9.

3B2

Work for problem 3(c)

total people in line = $700 + \int_0^t r(t)\,dt - 800t$

at time $t = 3$ because the rate of people boarding there overtakes the number arriving at the line

$800 + 700$

1,500 people are in line at time $t = 3$

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Work for problem 3(d)

$700 + \int_0^t r(t)\,dt - 800t$

END OF PART A OF SECTION II

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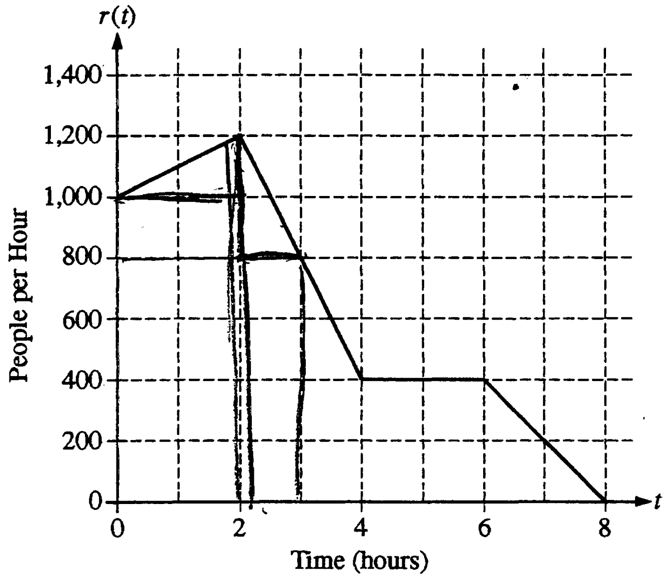
3C, 1



Work for problem 3(a)

$\int_0^3 r(t)\,dt$

$= \frac{2(200)}{2} + \frac{1(400)}{2} + 2(200) + 3(800)$

$=$ 3200 people arrive at the ride between $t=0$ and $t=3$

Work for problem 3(b)

$-\int_2^3 800\,d(t) + \int_2^3 r(t)\,dt$

$-800 + \frac{1(400)}{2} + 1(800)$

$= 200$

The number of people waiting in line to get on the ride is increasing because the rate at which people is decreasing.

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Continue problem 3 on page

3C2

Work for problem 3(c)

$\frac{dy}{dt}\left(\int_0^t r(t)dt - \int_0^t 800\,dt\right)$

$= \int_0^2 r(t)dt - \int_0^2 800\,(dt)$

$= 2{,}200 - 1600$

$= 600$

There are 600 people in line at time $t=2$.

At time 2, the line for the ride is the longest because $r(t)$ changes from + to −

Work for problem 3(d)

$0 = \int_0^t r(t)dt - \int_0^t 800\,dt$

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END OF PART A OF SECTION II
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AP® CALCULUS AB
2010 SCORING COMMENTARY

Question 3

Overview

The context for this problem was the line for an amusement-park ride. It was given that 700 people were in line when the ride began operation in the morning, and the rate $r(t)$, in people per hour, at which people join the ride was supplied via a piecewise-linear graph for $0 \le t \le 8$. It was also given that during the eight hours the ride is in operation, people move onto it at the rate of 800 people per hour, provided there are people waiting. Part (a) asked for the number of people arriving at the ride between times $t = 0$ and $t = 3$ hours. Students needed to obtain this value by computing $\int_0^3 r(t)\,dt$ geometrically from the supplied graph. Part (b) asked whether the length of the line was increasing or decreasing between times $t = 2$ and $t = 3$ hours. Students could determine this by comparing the rate $r(t)$ at which the line grows to 800 people per hour, the rate at which people move from the line onto the ride. Part (c) asked for the time t when the line was longest and the length of the line at that time. Students needed to recognize that the line is growing when $r(t) > 800$ and shrinking when $r(t) < 800,$ so that the line is at its longest during the one time ($t = 3$) when the graph of r decreases through the value $r = 800.$ The number of people waiting in line at that time is computed by subtracting the $3 \cdot 800 = 2400$ people that move from the line onto the ride during the 3 hours from the sum of the 700 people in line at time $t = 0$ and $\int_0^3 r(t)\,dt,$ the number of people joining the line between times $t = 0$ and $t = 3$ hours. Part (d) asked for an equation whose solution gives the earliest time t at which there were no longer people in line. This occurs when the number of people that have joined the line by time $t,$ $700 + \int_0^t r(s)\,ds,$ matches the number that have moved from the line to the ride, $800t.$

Sample: 3A
Score: 9

The student earned all 9 points. In part (c) the student's phrase "so the lineup will be shorter" was ignored.

Sample: 3B
Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the student's work is correct. In part (b) the student fails to justify "increasing." The student was required to make an explicit comparison of rates, rather than amounts. In part (c) the student earned the first 2 points but does not give a global argument for the justification. In part (d) the student earned the first 2 points.

Sample: 3C
Score: 4

The student earned 4 points: 2 points in part (a), no points in part (b), no points in part (c), and 2 points in part (d). In part (a) the student's work is correct. In part (b) the student fails to justify "increasing." In part (c) the student's work is incorrect. In part (d) the student earned the first point for an implicit $800t$ and the second point for the integral. The student's equation does not include the initial condition of 700, so the third point was not earned.